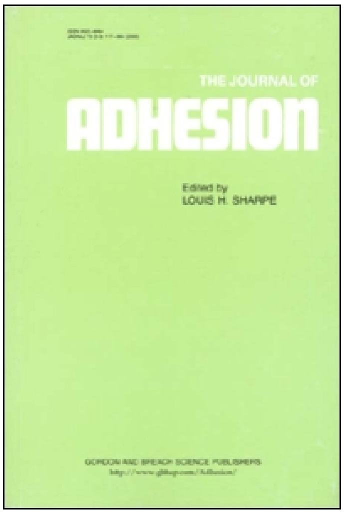

The Journal of Adhesion
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713453635

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To cite this Article Thamm, Frigyes(1976) 'Stress Distribution in Lap Joints With Partially Thinned Adherends', The Journal of Adhesion, 7: 4, 301 — 309

To link to this Article: DOI: 10.1080/00218467608075061

URL: http://dx.doi.org/10.1080/00218467608075061

J. Adhesion, 1976, Vol. 7, pp. 301–309

Printed in Scotland

Stress Distribution in Lap Joints With Partially Thinned Adherends

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Translated by W. C. Wake from a paper given in German at the VIth Conference, Intermetal Bond, organized by the Czechoslovak Scientific and Technical Society, SVTS, Bratislava. The paper appeared earlier in Periodica Polytechnica (Mechanical Engng.) vol. 18, nos. 2/3, 103–112 (1974) in substantially the same form.

(*Received June 24, 1975*)

The favourable strength properties of adhesive joints with thinned-out overlaps compared with those with simple overlap has long been established experimentally as well as theoretically.[1,2] Research previously carried out was concerned merely with the case of complete chamfering by which the end of the overlapped parts of the joint takes the form of a cutting edge (Figure 1). As the manufacture of the edge requires careful workmanship and to avoid accidents the handling of the edge must likewise be followed with great care, it is usual only partially to sharpen the faying parts so that the end of the

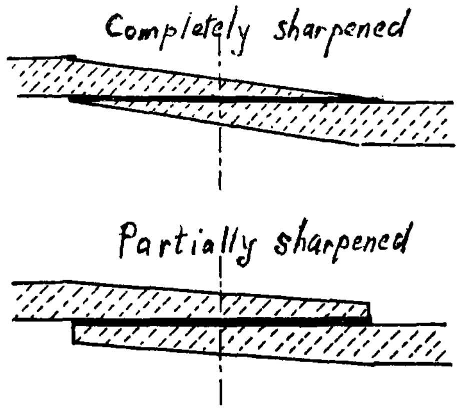

FIGURE 1 Completely and partially sharpened overlapped adhesive joints.

faying parts still show a terminal thickness. For the case of partial sharpening the results and calculations obtained for fully sharpened adherends are not generally valid or only in the limited sense. Thus the strength advantage of sharpened as compared with simple joints appears questionable. For this reason, investigations were undertaken of the shear stress distribution of partially sharpened adhesive joints as is communicated in the following paragraphs.

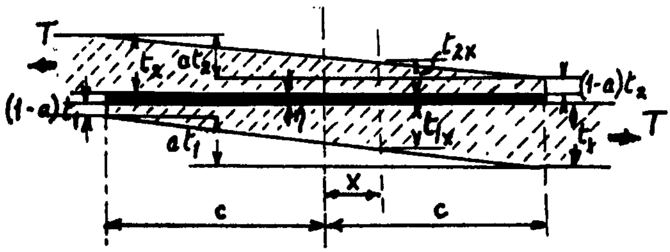

FIGURE 2 Nomenclature for partially sharpened adhesive joint.

The dimensions used for the investigation of partially sharpened adhesive joints are shown in Figure 2. The extent of the sharpening shown in Figure 2 is also characterized by the factor a. The thickness of the faying part at the point x is obtained from the remaining dimensions as

$$\left.\begin{aligned} t_{1x} &= t_1\left(1 - a\frac{c - x}{2c}\right) = \frac{t_1}{2c}(2c - ac + ax) \\ t_{2x} &= t_2\left(1 - a\frac{c + x}{2c}\right) = \frac{t_2}{2c}(2c - ac - ax) \end{aligned}\right\} \qquad (1)$$

The elastic modulus of both faying parts is the same and has the value E_L. The shear modulus of the adhesive when set is G_R. The externally applied tensile force T applied over 1 cm width of the joint amounts, corresponding to the notation of figure 2, at the point x for both faying parts, according to figure 3, to T_{1x} and T_{2x}. The equilibrium of the components of the force which act on the part to the right of the cross-section x of the adhesive joint, can be written down as follows:

$$T_{2x} = T - T_{1x} \qquad (2)$$

corresponding to Volkersen's calculation,[3] the deviation from the working axis of the resultant of T_{1x} and T_{2x} can be neglected compared with T. Consequently, the curvature taken up by the adherends remains and thus the stress components normal to the adhesive film tending to tear apart the two adherends are not considered.[6] Considering only the elastic deformations, the displacement of the length elements of the adherends from the length dx

in the x-direction amounts to U_{1x} and U_{2x} respectively. Consequently the extension of the two faying parts is given by

$$\left.\begin{aligned} \varepsilon_{1x} &= \frac{du_{1x}}{dx} = \frac{T_{1x}}{E_L t_{1x}} \\ \varepsilon_{2x} &= \frac{du_{2x}}{dx} = \frac{T_{2x}}{E_L t_{2x}} \end{aligned}\right\} \tag{3}$$

The equilibrium of forces acting on the single adherend element yields the following equations.

$$\left.\begin{aligned} dT_{1x} - \tau_x \, dx &= 0 \\ dT_{2x} + \tau_x \, dx &= 0 \end{aligned}\right\} \tag{4}$$

A shear stress distribution is assumed homogeneous through the thickness of the adhesive film so that the direction in which the shear stresses are extended is always parallel and correspondingly normal to the upper surface

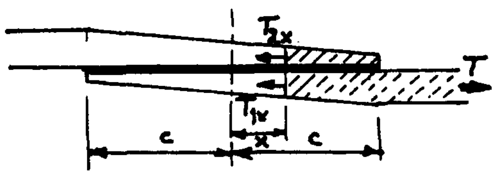

FIGURE 3 Forces along length of adherends at cross-section x.

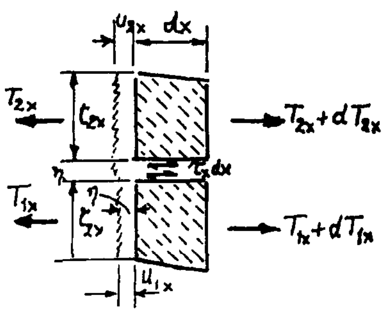

FIGURE 4 Deformation of the length elements in the x direction of the adherends of length dx at x and the forces acting on them.

of the adherend; the characterization of the direction of the shear stresses by two indices appears superfluous. The index x refers not to the direction of the shear stress but to x indicated on the section in Figures 2 and 3. The change of the right angle between the surfaces of an originally cubic element of volume of the adhesive film by elastic deformation is γ. The relation between the change of angle and the shear force τ_x can be written as

$$\gamma \simeq \tan\gamma = \frac{U_{1x} - U_{2x}}{\eta} = \frac{\tau_x}{G_R} \tag{5}$$

since the angle γ can, as a matter of fact, be taken as small. The first equation of group (4), placed in Eq. (5) yields

$$U_{1x} - U_{2x} = \frac{\eta}{G_R}\frac{dT_{1x}}{dx} \tag{6}$$

The validity of Eq. (6) depends on the assumption that the shear deformation of the adherends can be neglected. By analogy with the work of Goland and Reissner,[6] the range of validity can be given with a precision sufficient for practical application by the following equation in which G_L is the shear modulus of the adherends.

$$\frac{\eta}{G_R} > 10\,\frac{(t_1 + t_2)}{2G_L}\left(1 - \frac{a}{2}\right) \tag{6a}$$

If eq. (6) is differentiated again with respect to x and in the equation so obtained the equations of group (3) are substituted, one obtains

$$\frac{\eta}{G_R}\frac{d^2T_{1x}}{dx^2} = \frac{T_{1x}}{E_L t_{1x}} - \frac{T_{2x}}{E_L t_{2x}} \tag{7}$$

corresponding, by consideration of Eq. (2) to the following transformation:

$$\frac{d^2T_{1x}}{dx^2} = \frac{G_R}{\eta E_L}\left[\frac{1}{t_{1x}} + \frac{1}{t_{2x}}\right]T_{1x}\,\frac{G_R}{\eta E_L}\frac{T}{t_{2x}} \tag{8}$$

The values of t_{1x} and t_{2x} from Eq. (1) put into Eq. (8) yields,

$$\frac{d^2T_{1x}}{dx^2} = \frac{G_R 2c}{E_L\eta}\,\frac{t_1(2c - ac + ax) + t_2(2c - ac - ax)}{(2c - ac + ax)(2c - ac - ax)t_1t_2}\,T_{1x} - \frac{G_R}{E_L\eta}\,\frac{2c}{t_2(2c - ac + ax)}\,T. \tag{9}$$

This, by the introduction of the parameter $\mu = G_R/E_Lt_2$ and of the ratio of the strengths of the adherends $\omega = (t_1 + t_2)/t_1$, corresponds to

$$\frac{d^2T_{1x}}{dx^2} = \mu 2c\left\{\frac{1}{2c - ac - ax} + \frac{t_2}{t_1}\,\frac{1}{2c - ac + ax}\right\}T_{1x} - T\,\frac{\mu\, 2c}{2c - ac - ax} \tag{10}$$

The introduction of the dimensionless variables $\phi = T_{1x}/T$ and $\xi = x/c$, as well as the dimensionless stiffness number $m = G_RC^2/(E_Lt_2\eta)$ into Eq. (10) yields:

$$\frac{d^2\phi}{d\xi^2} = m\left\{\frac{1}{1 - \frac{1}{2}a - \frac{1}{2}a\xi} + \frac{\omega - 1}{1 - \frac{1}{2}a + \frac{1}{2}a\xi}\right\}\phi - \frac{m}{1 - \frac{1}{2}a - \frac{1}{2}a\xi}. \tag{11}$$

The mean value of the shear stress caused by the tensile force T amounts to

$$\tau_K = T/2c \tag{12}$$

The shear stress at the place x produces from Eq. (4),

$$\tau_x = \frac{dT_{1x}}{dx} \tag{13}$$

Eq. (13) divided by Eq. (12) on consideration of the meanings of ϕ and ξ, gives

$$\frac{\tau_x}{\tau_K} = \frac{2c}{T}\frac{dT_{1x}}{dx} = 2\,\frac{d\phi}{d\xi} \tag{14}$$

The differential Eq. (11) must, corresponding to Figure 1, utilize the following boundary conditions:

$$\left.\begin{array}{l}\text{for } x = +c \text{ corresponding to } \xi = +1,\ \phi = -1\\ \text{for } x = -c \text{ corresponding to } \xi = -1,\ \phi = 0\end{array}\right\} \tag{15}$$

As the analytical solution of the differential Eq. (11) is unknown, it must be solved by numerical methods. For this purpose the differential equation was transformed into a finite–difference equation. The range $-1 < \xi < +1$ was divided into $m = 100$ parts and the symbol $\chi = 2/n$ used, yielding[4] for the second derivative of ϕ on ξ as the ith part is approached ($\xi = \xi_i$)

$$\left(\frac{d^2\phi}{d\xi^2}\right)_{\xi i} = \frac{\phi_{i+1} - 2\phi_i + \phi_{i-1}}{\chi^2} \tag{16}$$

whereby Eq. (11) can be written in the following form

$$\frac{\phi_{i-1}}{\chi^2} - \phi_i\left[\frac{m}{1 - \frac{1}{2}a(1 - \xi_i)} + \frac{m(\omega - 1)}{1 - \frac{1}{2}a(1 - \xi_i)} + \frac{2}{\chi^2}\right] + \frac{\phi_{i+1}}{\chi^2} = \frac{-m}{1 - \frac{1}{2}a(1 + \xi_i)} \tag{17}$$

This equation, entered for each value of i, and allowing for the boundary conditions yield n algebraic equations with n unknowns. The solution of these equations ensues on a digital computer by means of the technique of matrix inversion for which, in cases such as this, a programme is conveniently available.[5] Eq. (11) and (17) are also valid for the completely sharpened case with the insertion of $a = 1$. This value for a put into Eq. (11) yields, after several transformations,

$$\frac{1}{2m}\frac{d^2\phi}{d\xi^2} = -\frac{1 - \phi}{1 - \xi} + \frac{(\omega - 1)\phi}{1 + \xi} \tag{18}$$

As the right hand side of Eq. (18) has an infinite value for the boundaries of the overlap ($\xi = -1$ as well as $\xi = +1$), Eq. (18) is unsuitable for numerical treatment. For this reason it was further transformed and yielded the following equation.

$$\frac{1-\xi^2}{2m}\frac{d^2\phi}{d\xi^2} = \phi\{\xi(2-\omega)+\omega\} - \{1+\xi\} \tag{19}$$

In the case where the lengths of the whole overlap are uniform, distribution of the shear stress corresponds fundamentally to a solution of Eq. (19) in the following form

$$\phi = \tfrac{1}{2}\xi + C \tag{20}$$

Here, C is an integration constant and if put into Eq. (19) the following identity is obtained:

$$0 = (\tfrac{1}{2}\xi + C)\{\xi(2-\omega)+\omega\} - (1+\omega) \tag{21}$$

from which, as it must be valid for all values of ξ, for the coefficients of the various powers of ξ yield the following equation:

Coefficients of ξ^2: $\quad 0 = \tfrac{1}{2}(2-\omega)$ (22a)

Coefficients of ξ: $\quad 0 = \tfrac{1}{2}\omega + C(2-\omega) - 1$ (22b)

Not containing ξ: $\quad 0 = -C\omega + 1$ (22c)

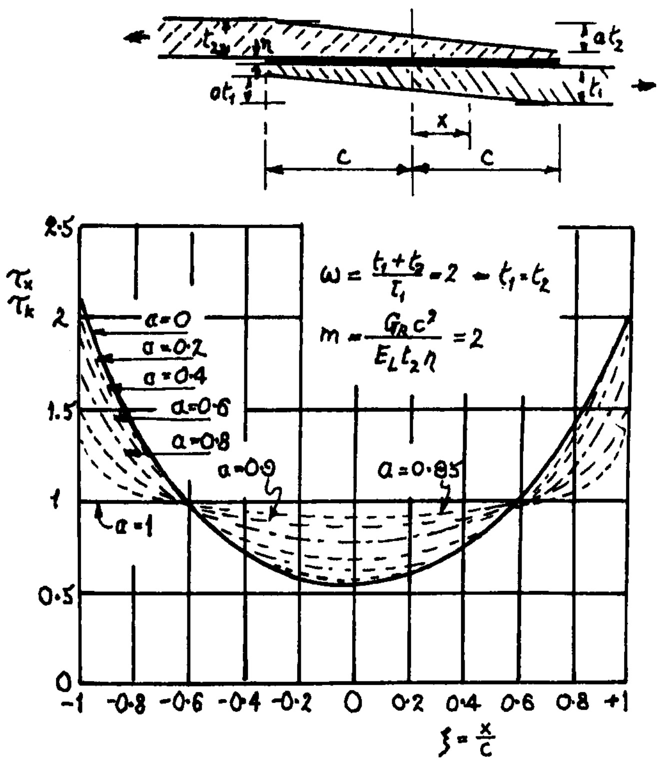

FIGURE 5

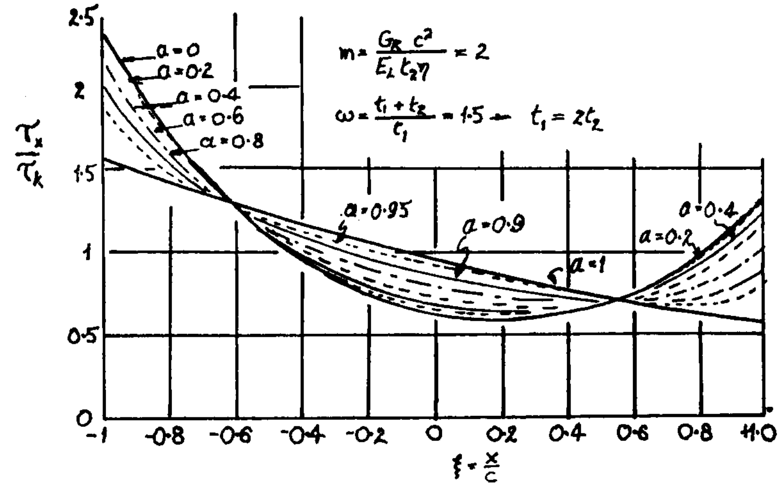

FIGURE 6

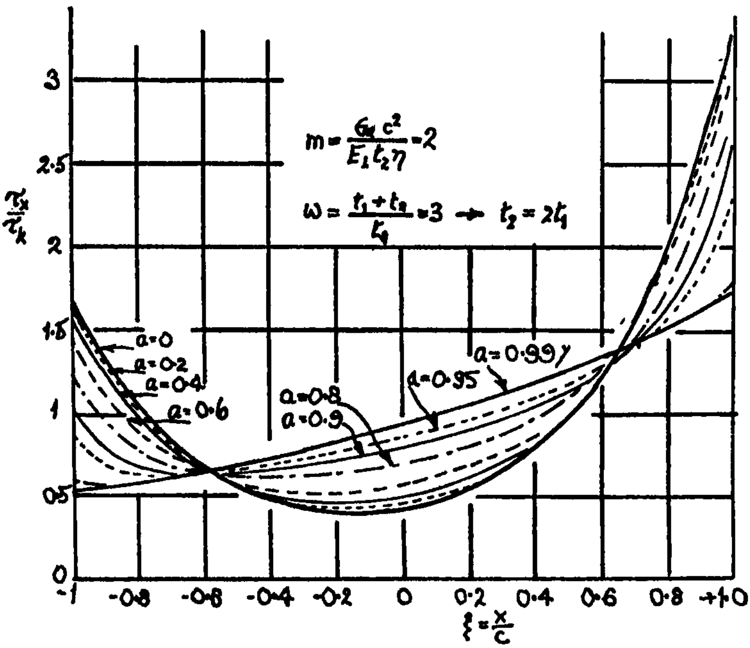

FIGURE 7

FIGURES 5, 6 and 7 Distribution of the shear stress τ_x as a ratio of the mean shear in the adhesive along the length of the overlap, $2C$, for a stiffness number $m = 2$ and various amounts of sharpening, a.

Equations (22) lead to $\omega = 2$ and $C = 1/\omega = \frac{1}{2}$; that is the shear stress only appears uniformly distributed along the whole length of the overlap in the case of adherends of equal thickness. This agrees fully with von Vocke's derivation.[2] For adherends of different thickness, that is $\omega \neq 2$, the distribution $\phi(\xi)$ will be determined through one of Eq. (16) or (17) by the corresponding numerical methods. ·

The distribution of τ_x/τ_k obtained from the solution of Eq. (11) and (19) allowing for the boundary conditions (15) for $m = 2$ and $\omega = 1.5$, 2 and 3 is shown in Figures 5, 6 and 7. The distribution of shear stress has been sketched in the three figures of various degrees of sharpening, i.e. values of a. The highest value of the shear stress in the adhesive film for $m = 2$, and $\omega = 1.5$, 2 and 3 and its dependence on the amount, a, of the sharpening is shown in Figure 8.

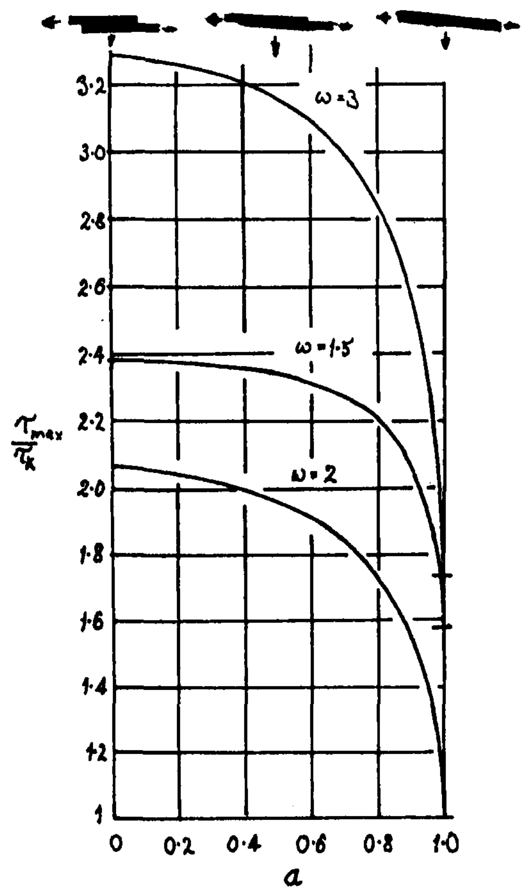

FIGURE 8 *Ratio of peak value τ_{max} to mean value τ_K of the shear stress as a function of* degree of sharpening, a, for $m = 2$ and $\omega = 1.5$, 2 and 3.

As is apparent from Figure 8, the value of the peak shear diminishes only slightly with partial sharpening by about $a = 0.5$ compared with a simple overlap. Only from about $a = 0.8$ is a rapid lessening of the peak shear stress to be observed as a increases.

The results shown in Figures 5 to 7 are also applicable in the case where the limits of validity (6a) are exceeded if the shear deformation of the adherends corresponding to the assumptions of Demarkles[7] and Adams–Peppiatt[8] is considered and the dimensionless stiffness number m is written in the following form:

$$m = \frac{G_R C^2}{E_L t_2 \left\{ \eta + \frac{G_R}{2G_L}(t_1 + t_2)(1 - \frac{1}{2}a) \right\}} \tag{23}$$

The following conclusions for practical adhesives technology can be based on the results of the calculations.

a) Sharpening of the adherends of overlapped adhesive joints yields a substantial increase in strength only when the sharpening is complete; that is the adherends are sharpened as far as possible to an edge. Partial sharpening to about half the original thickness is practically valueless from the strength viewpoint.

b) As the highest value of the shear stress climbs very steeply with falling values of a in the neighbourhood of $a = 1$, the slightest damage to the edge of a completely sharpened adherend can cause a considerable increase of the peak stress and thereby lead to lowering of tensile shear strength of the joint.

c) If, on grounds of difficulty of manufacture, the sharpening can be no more than $a = 0.9$, the reduction in the value of the peak stress, compared with simple overlapping, is only a fraction of that to be obtained with complete sharpening. It is questionable whether, in this case, the increased expenditure required for the sharpening is compensated by the not very great rise in the tensile shear stress thus obtained.

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